

Lesson 1 Impulse and Momentum

Mastering Concepts

1. Can a bullet have the same momentum as a truck? Explain.

Yes, for a bullet to have the same momentum as a truck, it must have a higher velocity because the two masses are not the same.

$$m_{\text{bullet}}v_{\text{bullet}} = m_{\text{truck}}v_{\text{truck}}$$

2. During a baseball game, a pitcher throws a curve ball to the catcher. Assume that the speed of the ball does not change in flight.

- a. Which player exerts the larger impulse on the ball?

The pitcher and the catcher exert the same amount of impulse on the ball, but the two impulses are in opposite directions.

- b. Which player exerts the larger force on the ball?

The catcher exerts the larger force on the ball because the time interval over which the force is exerted is smaller.

3. Newton's second law of motion states that if no net force is exerted on a system, no acceleration is possible. Does it follow that no change in momentum can occur?

No net force on the system means no net impulse on the system and no net change in momentum. However, individual parts of the system may have a change in momentum as long as the net change in momentum is zero.

4. Why are cars made with bumpers that can be pushed in during a crash?

Cars are made with bumpers that compress during a crash to increase the time of a collision, thereby reducing the force.

5. An ice-skater is doing a spin.
- How can the skater's angular momentum be changed?
by applying an external torque
 - How can the skater's angular velocity be changed without changing the angular momentum?
by changing the moment of inertia

Mastering Problems

6. **Golf** Rocío strikes a 0.058-kg golf ball with a force of 272 N and gives it a velocity of 62.0 m/s. How long was Rocío's club in contact with the ball? (Level 1)

$$\begin{aligned}\Delta t &= \frac{m\Delta v}{F} \\ &= \frac{(0.058 \text{ kg})(62.0 \text{ m/s})}{272 \text{ N}} \\ &= 0.013 \text{ s}\end{aligned}$$

7. A 0.145-kg baseball is pitched at 42 m/s. The batter hits the ball horizontally toward the pitcher at 58 m/s. (Level 2)

- a. Find the change in momentum of the ball.

Take the direction of the pitch to be positive.

$$\begin{aligned}\Delta p &= mv_f - mv_i \\ &= m(v_f - v_i) \\ &= (0.145 \text{ kg})(-58 \text{ m/s} - 42 \text{ m/s}) \\ &= -14 \text{ kg}\cdot\text{m/s}\end{aligned}$$

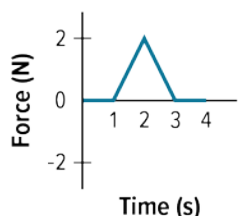
- b. If the ball and the bat are in contact for 4.6×10^{-4} s, what is the average force during contact?

$$\begin{aligned}F\Delta t &= \Delta p \\ F &= \frac{\Delta p}{\Delta t} \\ &= \frac{m(v_f - v_i)}{\Delta t} \\ &= \frac{(0.145 \text{ kg})(-58 \text{ m/s} - (42 \text{ m/s}))}{4.6 \times 10^{-4} \text{ s}} \\ &= -3.2 \times 10^4 \text{ N}\end{aligned}$$

8. A 0.150-kg ball, moving in the positive direction at 12 m/s, is acted on by the impulse illustrated in the graph. What is the ball's speed at 4.0 s? (Level 2)

Module 9 continued

Force v. Time



$$F\Delta t = m\Delta v$$

$$\text{Area of graph} = m\Delta v$$

$$\frac{1}{2}(2.0 \text{ N})(2.0 \text{ s}) = m(v_f - v_i)$$

$$2.0 \text{ N}\cdot\text{s} = (0.150 \text{ kg})(v_f - 12 \text{ m/s})$$

$$v_f = \frac{2.0 \text{ kg}\cdot\text{m/s}}{0.150 \text{ kg}} + 12 \text{ m/s} = 25 \text{ m/s}$$

9. **Bowling** A force of 186 N acts on a 7.3-kg bowling ball for 0.40 s. What is the ball's change in momentum? What is its change in velocity? (Level 1)

$$\Delta p = F\Delta t$$

$$= (186 \text{ N})(0.40 \text{ s})$$

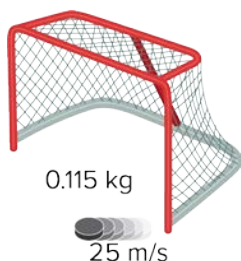
$$= 74 \text{ N}\cdot\text{s} = 74 \text{ kg}\cdot\text{m/s}$$

$$\Delta v = \frac{\Delta p}{m} = \frac{F\Delta t}{m}$$

$$= \frac{(186 \text{ N})(0.40 \text{ s})}{7.3 \text{ kg}}$$

$$= 1.0 \times 10^1 \text{ m/s}$$

10. **Hockey** A hockey puck has a mass of 0.115 kg and strikes the pole of the net at 37 m/s. It bounces off in the opposite direction at 25 m/s as shown. (Level 2)



- a. What is the impulse on the puck?

$$F\Delta t = m(v_f - v_i)$$

$$= (0.115 \text{ kg})(-25 \text{ m/s} - 37 \text{ m/s})$$

$$= -7.1 \text{ kg}\cdot\text{m/s}$$

- b. If the collision takes 5.0×10^{-4} s, what is the average force on the puck?

$$F\Delta t = m(v_f - v_i)$$

$$F = \frac{m(v_f - v_i)}{\Delta t}$$

$$= \frac{(0.115 \text{ kg})(-25 \text{ m/s} - 37 \text{ m/s})}{5.0 \times 10^{-4} \text{ s}}$$

$$= -1.4 \times 10^4 \text{ N}$$

11. A 5500-kg freight truck accelerates from 4.2 m/s to 7.8 m/s in 15.0 s by the application of a constant force. (Level 1)

- a. What change in momentum occurs?

$$\Delta p = m\Delta v = m(v_f - v_i)$$

$$= (5500 \text{ kg})(7.8 \text{ m/s} - 4.2 \text{ m/s})$$

$$= 2.0 \times 10^4 \text{ kg}\cdot\text{m/s}$$

- b. How large a force is exerted?

$$F = \frac{\Delta p}{\Delta t} = \frac{m(v_f - v_i)}{\Delta t}$$

$$= \frac{(5500 \text{ kg})(7.8 \text{ m/s} - 4.2 \text{ m/s})}{15.0 \text{ s}}$$

$$= 1.3 \times 10^3 \text{ N}$$

12. In a ballistics test at the police department, Officer Rios fires a 6.0-g bullet at 350 m/s into a container that stops it in 1.8 ms. What is the average force that stops the bullet? (Level 1)

$$F = \frac{\Delta p}{\Delta t} = \frac{m(v_f - v_i)}{\Delta t}$$

$$= \frac{(0.0060 \text{ kg})(0.0 \text{ m/s} - 350 \text{ m/s})}{1.8 \times 10^{-3} \text{ s}}$$

$$= -1.2 \times 10^3 \text{ N}$$

13. **Volleyball** A 0.24-kg volleyball approaches Tina with a velocity of 3.8 m/s. Tina bumps the ball, giving it a speed of 2.4 m/s but in the opposite direction. What average force did she apply if the interaction time between her hands and the ball was 0.025 s? (Level 1)

$$F = \frac{m\Delta v}{\Delta t}$$

$$= \frac{(0.24 \text{ kg})(-2.4 \text{ m/s} - 3.8 \text{ m/s})}{0.025 \text{ s}}$$

$$= -6.0 \times 10^1 \text{ N}$$

Module 9 continued

14. Before a collision, a 25-kg object was moving at 112 m/s. Find the impulse that acted on the object if, after the collision, it moved at the following velocities. (Level 1)

a. +8.0 m/s

$$\begin{aligned} F\Delta t &= m\Delta v = m(v_f - v_i) \\ &= (25 \text{ kg})(8.0 \text{ m/s} - 12 \text{ m/s}) \\ &= -1.0 \times 10^2 \text{ kg}\cdot\text{m/s} \end{aligned}$$

b. -8.0 m/s

$$\begin{aligned} F\Delta t &= m\Delta v = m(v_f - v_i) \\ &= (25 \text{ kg})(-8.0 \text{ m/s} - 12 \text{ m/s}) \\ &= -5.0 \times 10^2 \text{ kg}\cdot\text{m/s} \end{aligned}$$

15. **Baseball** A 0.145-kg baseball is moving at 35 m/s when it is caught by a player. (Level 2)

a. Find the change in momentum of the ball.

$$\begin{aligned} \Delta p &= m(v_f - v_i) \\ &= (0.145 \text{ kg})(0.0 \text{ m/s} - 35 \text{ m/s}) \\ &= -25.1 \text{ kg}\cdot\text{m/s} \end{aligned}$$

b. If the ball is caught with the mitt held in a stationary position so that the ball stops in 0.050 s, what is the average force exerted on the ball?

$$\begin{aligned} \Delta p &= F_{\text{avg}} \Delta t \\ F_{\text{avg}} &= \frac{\Delta p}{\Delta t} = \frac{m(v_f - v_i)}{\Delta t} \\ &= \frac{(0.145 \text{ kg})(0.0 \text{ m/s} - 35 \text{ m/s})}{0.050 \text{ s}} \\ &= -1.0 \times 10^2 \text{ N} \end{aligned}$$

c. If, instead, the mitt is moving backward so that the ball takes 0.500 s to stop, what is the average force exerted by the mitt on the ball?

$$\begin{aligned} \Delta p &= F_{\text{avg}} \Delta t \\ F_{\text{avg}} &= \frac{\Delta p}{\Delta t} = \frac{m(v_f - v_i)}{\Delta t} \\ &= \frac{(0.145 \text{ kg})(0.0 \text{ m/s} - 35 \text{ m/s})}{0.500 \text{ s}} \\ &= -1.0 \times 10^1 \text{ N} \end{aligned}$$

16. **Ranking Task** Rank the following objects according to the amount of momentum they have, from least to greatest. Specifically indicate any ties. (Level 1)

20. A nitrogen molecule with a mass of 4.7×10^{-26} kg, moving at 550 m/s, strikes the wall of a container and bounces back at the same speed. (Level 2)

Object A:

mass 2.5 kg, velocity 1.0 m/s east

Object B:

mass 3.0 kg, velocity 0.9 m/s west

Object C:

mass 3.0 kg, velocity 1.2 m/s west

Object D:

mass 4.0 kg, velocity 0.5 m/s north

Object E:

mass 4.0 kg, velocity 0.9 m/s east

$$p = mv$$

$$p_A = (2.5 \text{ kg})(1.0 \text{ m/s}) = 2.5 \times 10^1 \text{ kg}\cdot\text{m/s}^2$$

$$p_B = (3.0 \text{ kg})(0.90 \text{ m/s}) = 2.7 \times 10^1 \text{ kg}\cdot\text{m/s}^2$$

$$p_C = (3.0 \text{ kg})(1.2 \text{ m/s}) = 3.6 \times 10^1 \text{ kg}\cdot\text{m/s}^2$$

$$p_D = (4.0 \text{ kg})(0.50 \text{ m/s}) = 2.0 \times 10^1 \text{ kg}\cdot\text{m/s}^2$$

$$p_E = (4.0 \text{ kg})(0.90 \text{ m/s}) = 3.6 \times 10^1 \text{ kg}\cdot\text{m/s}^2$$

$$D < A < B < C = E$$

17. **Hockey** A hockey player makes a slap shot, exerting a constant force of 30.0 N on the puck for 0.16 s. What is the magnitude of the impulse given to the puck? (Level 1)

$$\begin{aligned} F\Delta t &= (30.0 \text{ N})(0.16 \text{ s}) \\ &= 4.8 \text{ N}\cdot\text{s} \end{aligned}$$

18. **Skateboarding** Your brother's mass is 35.6 kg, and he has a 1.3-kg skateboard. What is the combined momentum of your brother and his skateboard if they are moving at 9.50 m/s? (Level 1)

$$\begin{aligned} p &= mv \\ &= (m_{\text{boy}} + m_{\text{board}})v \\ &= (35.6 \text{ kg} + 1.3 \text{ kg})(9.50 \text{ m/s}) \\ &= 3.5 \times 10^2 \text{ kg}\cdot\text{m/s} \end{aligned}$$

19. A hockey puck has a mass of 0.115 kg and is at rest. A hockey player makes a shot, exerting a constant force of 30.0 N on the puck for 0.16 s. With what speed does it head toward the goal? (Level 1)

$$\begin{aligned} F\Delta t &= m\Delta v = m(v_f - v_i) \\ \text{where } v_i &= 0 \\ \text{Thus } v_f &= \frac{F\Delta t}{m} \\ &= \frac{(30.0 \text{ N})(0.16 \text{ s})}{0.115 \text{ kg}} \\ &= 42 \text{ m/s} \end{aligned}$$

Module 9 continued

- a. What is the molecule's impulse on the wall?

$$\begin{aligned} F\Delta t &= m(v_f - v_i) \\ &= (4.7 \times 10^{-26} \text{ kg})(-550 \text{ m/s} - 550 \text{ m/s}) \\ &= -5.2 \times 10^{-23} \text{ kg}\cdot\text{m/s} \end{aligned}$$

The impulse the wall delivers to the molecule is $-5.2 \times 10^{-23} \text{ kg}\cdot\text{m/s}$.

The impulse the molecule delivers to the wall is $+5.2 \times 10^{-23} \text{ kg}\cdot\text{m/s}$.

- b. If there are 1.5×10^{23} of the collisions each second, what is the average force on the wall?

$$F\Delta t = m(v_f - v_i)$$

$$F = \frac{m(v_f - v_i)}{\Delta t}$$

For all the collisions, the force is

$$\begin{aligned} F_{\text{total}} &= (1.5 \times 10^{23}) \frac{m(v_f - v_i)}{\Delta t} \\ &= (1.5 \times 10^{23}) \frac{(4.7 \times 10^{-26} \text{ kg})(-550 \text{ m/s} - 550 \text{ m/s})}{1.0 \text{ s}} \\ &= -7.8 \text{ N} \end{aligned}$$

The force on the molecules is -7.8 N .

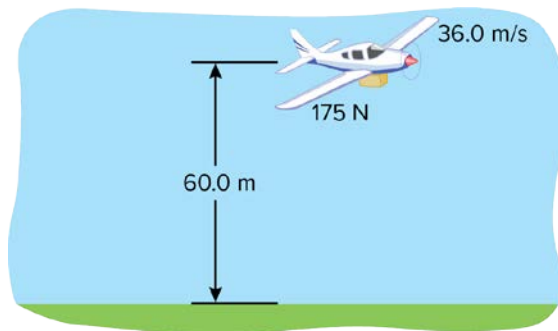
The force on the wall is $+7.8 \text{ N}$.

21. **Rockets** Small rockets are used to slightly adjust the speeds of spacecraft. A rocket with a thrust of 35 N is fired to change a 72,000-kg spacecraft's speed by 63 cm/s. For how long should it be fired? (Level 3)

$$F\Delta t = m\Delta v$$

$$\begin{aligned} \text{so, } \Delta t &= \frac{m\Delta v}{F} \\ &= \frac{(72,000 \text{ kg})(0.63 \text{ m/s})}{35 \text{ N}} \\ &= 1.3 \times 10^3 \text{ s, or 22 min} \end{aligned}$$

22. An animal rescue plane flying due east at 36.0 m/s drops a 175-N bale of hay from an altitude of 60.0 m as shown. What is the momentum of the bale the moment before it strikes the ground? Give both magnitude and direction. (Level 3)



Module 9 continued

First use projectile motion to find the velocity of the bale.

$$p = mv$$

To find v , consider the horizontal and vertical components.

$$v_x = 36.0 \text{ m/s}$$

$$v_y^2 = v_{iy}^2 + 2xg = 2xg$$

Thus,

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_x^2 + 2xg}$$

The momentum, then, is

$$\begin{aligned} p &= \frac{F_g v}{g} = \frac{F_g \sqrt{v_x^2 + 2xg}}{g} \\ &= \frac{(175 \text{ N}) \sqrt{(36.0 \text{ m/s})^2 + (2)(60.0 \text{ m})(9.8 \text{ N/kg})}}{9.8 \text{ N/kg}} = 8.9 \times 10^2 \text{ kg}\cdot\text{m/s} \end{aligned}$$

The angle from the horizontal is

$$\begin{aligned} \theta &= \tan^{-1} \left(\frac{v_y}{v_x} \right) = \tan^{-1} \left(\frac{\sqrt{2xg}}{v_x} \right) \\ &= \tan^{-1} \left(\frac{\sqrt{(2)(60.0 \text{ m})(9.8 \text{ N/kg})}}{36.0 \text{ m/s}} \right) \\ &= 44^\circ \end{aligned}$$

- 23. Accident** A car moving at 10.0 m/s crashes into a barrier and stops in 0.050 s. There is a 20.0-kg child in the car. Assume that the child's velocity is changed by the same amount as that of the car, and in the same time period. (Level 3)

- a. What is the impulse needed to stop the child?

$$\begin{aligned} F\Delta t &= m\Delta v = m(v_f - v_i) \\ &= (20.0 \text{ kg})(0.0 \text{ m/s} - 10.0 \text{ m/s}) \\ &= -2.00 \times 10^2 \text{ kg}\cdot\text{m/s} \end{aligned}$$

- b. What is the average force on the child?

$$\begin{aligned} F\Delta t &= m\Delta v = m(v_f - v_i) \\ F &= \frac{m(v_f - v_i)}{\Delta t} \\ &= \frac{(20.0 \text{ kg})(0.0 \text{ m/s} - 10.0 \text{ m/s})}{0.050 \text{ s}} \\ &= -4.0 \times 10^3 \text{ N} \end{aligned}$$

- c. What is the approximate mass of an object whose weight equals the force in part b?

$$F_g = mg$$

$$\begin{aligned} m &= \frac{F_g}{g} = \frac{4.0 \times 10^3 \text{ N}}{9.8 \text{ N/kg}} \\ &= 4.1 \times 10^2 \text{ kg} \end{aligned}$$

- d. Could you lift such a weight with your arm?

No.

- e. Why is it advisable to use a proper restraining seat rather than hold a child on your lap?

You would not be able to protect a child on your lap in the event of a collision.

- 24. Reverse Problem** Write a physics problem with real-life objects for which the following equation would be part of the solution: (Level 2)

$$F = \frac{(1.3 \text{ kg})(20.0 \text{ cm/s} - 0.0 \text{ cm/s})}{0.55 \text{ s}}$$

Answers will vary, but a correct form of the answer is, "In a game of croquet, a 1.3-kg ball is struck by a mallet that makes contact with the ball for 0.55 s. The ball, initially at rest, is given a speed of 20.0 cm/s. What average force did the club exert on the ball?"

Lesson 2 Conservation of Momentum

Mastering Concepts

25. What is meant by “an isolated system”?

An isolated system has no external forces acting on it.

26. A spacecraft in outer space increases its velocity by firing its rockets. How can hot gases escaping from its rocket engine change the velocity of the craft when there is nothing in space for the gases to push against?

Momentum is conserved. The change in momentum of gases in one direction must be balanced by an equal change in momentum of the spacecraft in the opposite direction.

27. A cue ball travels across a pool table and collides with the stationary eight ball. The two balls have equal masses. After the collision, the cue ball is at rest. What must be true regarding the speed of the eight ball?

If you define the two balls as a system, the eight ball must be moving with the same velocity that the cue ball had just before the collision.

28. Consider a ball falling toward Earth.

- a. Why is the momentum of the ball not conserved?

The momentum of a falling ball is not conserved because a net external force, gravity, is acting on it.

- b. In what system that includes the falling ball is the momentum conserved?

One such system in which total momentum is conserved includes the ball plus Earth.

29. A falling basketball hits the floor. Just before it hits, the momentum is in the downward direction, and after it hits the floor, the momentum is in the upward direction.

- a. Why isn't the momentum of the basketball conserved even though the bounce is a collision?

The floor is outside the system, so it exerts an external force, and therefore, an impulse on the ball.

- b. In what system is the momentum conserved?

Momentum is conserved in the system of ball plus Earth.

30. Only an external force can change the momentum of a system. Explain how the internal force of a car's brakes brings the car to a stop.

The external force of a car's brakes can bring the car to a stop by stopping the wheels and allowing the external frictional force of the road against the tires to stop the car. If there is no friction—for example, if the road is icy—then there is no external force and the car does not stop.

31. Children's playgrounds often have circular-motion rides. How could a child change the angular momentum of such a ride as it is turning?

The child would have to exert a torque on it. He or she could stand next to it and exert a force tangential to the circle on the handles as they go past. He or she also could run at the side and jump onboard.

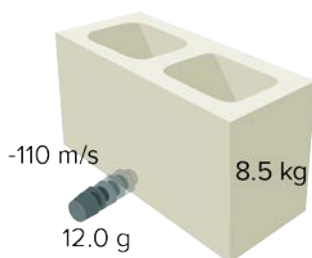
32. **Problem Posing** Complete this problem so that it can be solved using conservation of momentum: “Armando, mass 60.0 kg, is at the ice-skating rink ...”

Answers will vary. A possible form of the correct answer is, “... skating at a speed of 4.3 m/s when he collides head-on with Gabe, mass 50.0 kg, who is skating in the opposite direction at 2.7 m/s. The two stick together. If Armando and Gabe are a closed, isolated system, what is their final velocity?”

Module 9 continued

Mastering Problems

33. A 12.0-g rubber bullet travels at a forward velocity of 150 m/s, hits a stationary 8.5-kg concrete block resting on a frictionless surface, and ricochets in the opposite direction with a velocity of -110 m/s as shown. How fast will the concrete block be moving? (Level 1)



Define the bullet and the block as a closed, isolated system.

$$m_C v_{Ci} + m_D v_{Di} = m_C v_{Cf} + m_D v_{Df}$$

$$v_{Df} = \frac{m_C v_{Ci} + m_D v_{Di} - m_C v_{Cf}}{m_D}$$

Because the block is initially at rest, this becomes

$$v_{Df} = \frac{m_C (v_{Ci} - v_{Cf})}{m_D} = \frac{(0.0120 \text{ kg})(150 \text{ m/s} - (-110 \text{ m/s}))}{8.5 \text{ kg}} = 0.37 \text{ m/s}$$

34. **Football** A 95-kg fullback, running at 8.2 m/s, collides in midair with a 128-kg defensive tackle moving in the opposite direction. Both players end up with zero speed. (Level 1)

- a. Identify the before and after situations, and draw a diagram of both.

Define the two players as a closed, isolated system.

Before: $m_{FB} = 95 \text{ kg}$

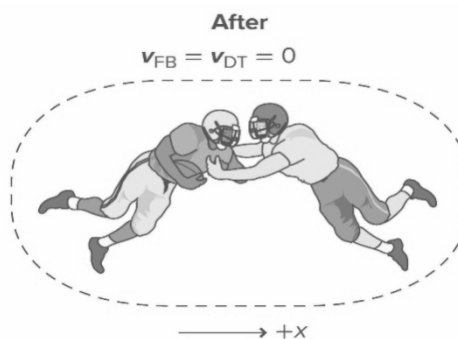
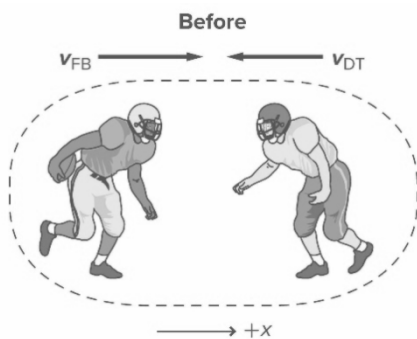
$v_{FB} = 8.2 \text{ m/s}$

$m_{DT} = 128 \text{ kg}$

$v_{DT} = ?$

After: $m = 223 \text{ kg}$

$v_f = 0 \text{ m/s}$



- b. What was the fullback's momentum before the collision?

$$\begin{aligned} p_{FB} &= m_{FB} v_{FB} = (95 \text{ kg})(8.2 \text{ m/s}) \\ &= 7.8 \times 10^2 \text{ kg}\cdot\text{m/s} \end{aligned}$$

- c. What was the change in the fullback's momentum?

$$\begin{aligned} \Delta p_{FB} &= p_f - p_{FB} \\ &= 0 - p_{FB} = -7.8 \times 10^2 \text{ kg}\cdot\text{m/s} \end{aligned}$$

Module 9 continued

- d. What was the change in the defensive tackle's momentum?

$$+7.8 \times 10^2 \text{ kg}\cdot\text{m/s}$$

- e. What was the defensive tackle's original momentum?

$$-7.8 \times 10^2 \text{ kg}\cdot\text{m/s}$$

- f. How fast was the defensive tackle moving originally?

$$m_{DT}v_{DT} = -7.8 \times 10^2 \text{ kg}\cdot\text{s}$$

So,

$$v_{DT} = \frac{-7.8 \times 10^2 \text{ kg}\cdot\text{s}}{128 \text{ kg}} = -6.1 \text{ m/s}$$

35. Marble C, with mass 5.0 g, moves at a speed of 20.0 cm/s. It collides with a second marble, D, with mass 10.0 g, moving at 10.0 cm/s in the same direction. After the collision, marble C continues with a speed of 8.0 cm/s in the same direction. (Level 1)

- a. Sketch the situation, and identify the system. Identify the before and after situations, and set up a coordinate system.

Define the marbles as a closed, isolated system.

Before: $m_C = 5.0 \text{ g}$

$$m_D = 10.0 \text{ g}$$

$$v_{Ci} = 20.0 \text{ cm/s}$$

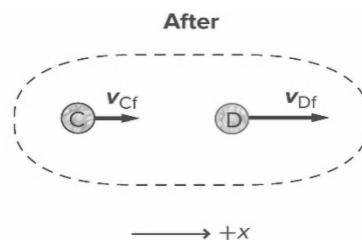
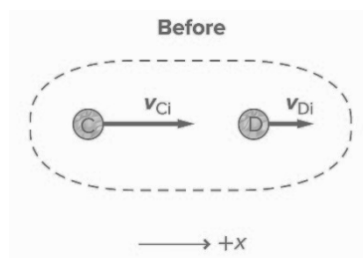
$$v_{Di} = 10.0 \text{ cm/s}$$

After: $m_C = 5.0 \text{ g}$

$$m_D = 10.0 \text{ g}$$

$$v_{Cf} = 8.0 \text{ cm/s}$$

$$v_{Df} = ?$$



- b. Calculate the marbles' momentums before the collision.

$$\begin{aligned} m_C v_{Ci} &= (5.0 \times 10^{-3} \text{ kg})(0.200 \text{ m/s}) \\ &= 1.0 \times 10^{-3} \text{ kg}\cdot\text{m/s} \end{aligned}$$

$$\begin{aligned} m_D v_{Di} &= (1.00 \times 10^{-2} \text{ kg})(0.100 \text{ m/s}) \\ &= 1.0 \times 10^{-3} \text{ kg}\cdot\text{m/s} \end{aligned}$$

- c. Calculate the momentum of marble C after the collision.

$$\begin{aligned} m_C v_{Cf} &= (5.0 \times 10^{-3} \text{ kg})(0.080 \text{ m/s}) \\ &= 4.0 \times 10^{-4} \text{ kg}\cdot\text{m/s} \end{aligned}$$

- d. Calculate the momentum of marble D after the collision.

$$p_{Ci} + p_{Di} = p_{Cf} + p_{Df}$$

$$p_{Df} = p_{Ci} = p_{Di} + p_{Cf}$$

$$\begin{aligned} &= 1.00 \times 10^{-3} \text{ kg}\cdot\text{m/s} + 1.00 \times 10^{-3} \text{ kg}\cdot\text{m/s} - 4.0 \times 10^{-4} \text{ kg}\cdot\text{m/s} \\ &= 1.6 \times 10^{-3} \text{ kg}\cdot\text{m/s} \end{aligned}$$

Module 9 continued

- e. What is the speed of marble D after the collision?

$$\begin{aligned}
 p_{Df} &= m_D v_{Df} \\
 \text{so, } v_{Df} &= \frac{p_{Df}}{m_D} \\
 &= \frac{1.6 \times 10^{-3} \text{ kg g m/s}}{1.00 \times 10^{-2} \text{ kg}} \\
 &= 1.6 \times 10^{-1} \text{ m/s} \\
 &= 0.16 \text{ m/s} \\
 &= 16 \text{ cm/s}
 \end{aligned}$$

36. Two lab carts are pushed together with a spring mechanism compressed between them. Upon being released, the 5.0-kg cart repels with a velocity of 0.12 m/s in one direction, while the 2.0-kg cart goes in the opposite direction. What is the velocity of the 2.0-kg cart? (Level 1)

Define the two carts from the moment they are released as a closed, isolated system.

$$\begin{aligned}
 m_1 v_i &= -m_2 v_f \\
 v_f &= \frac{m_1 v_i}{-m_2} \\
 &= \frac{(5.0 \text{ kg})(0.12 \text{ m/s})}{-(2.0 \text{ kg})} \\
 &= -0.30 \text{ m/s}
 \end{aligned}$$

37. A 50.0-g projectile is launched with a horizontal velocity of 647 m/s from a 4.65-kg launcher moving in the same direction at 2.00 m/s. What is the launcher's velocity after the launch? (Level 1)

Define the launcher and the projectile as a closed, isolated system.

$$\begin{aligned}
 p_{Ci} + p_{Di} &= p_{Cf} + p_{Df} \\
 m_C v_{Ci} + m_D v_{Di} &= m_C v_{Cf} + m_D v_{Df} \\
 \text{so, } v_{Df} &= \frac{m_C v_{Ci} + m_D v_{Di} - m_C v_{Cf}}{m_D}
 \end{aligned}$$

Assuming that the projectile, C, is launched in the direction of the launcher, D, motion,

$$\begin{aligned}
 v_{Df} &= \frac{(0.0500 \text{ kg})(2.00 \text{ m/s}) + (4.65 \text{ kg})(2.00 \text{ m/s}) - (0.500 \text{ kg})(647 \text{ m/s})}{4.65 \text{ kg}} \\
 &= -4.94 \text{ m/s, or } 4.94 \text{ m/s backwards}
 \end{aligned}$$

38. **Skateboarding** Kofi, with mass 42.00 kg, is riding a skateboard with a mass of 2.00 kg and traveling at 1.20 m/s. Kofi jumps off, and the skateboard stops dead in its tracks. In what direction and with what velocity did he jump? (Level 2)

Define the skateboard and Kofi as a closed, isolated system.

$$\begin{aligned}
 (m_K + m_S) v_i &= m_K v_{Kf} + m_S v_{Sf} \\
 \text{where } v_{Sf} &= 0 \text{ and } v_{Ki} = v_{Si} = v_i \\
 v_{Kf} &= \frac{(m_K + m_S) v_i}{m_K} \\
 &= \frac{(42.00 \text{ kg} + 2.00 \text{ kg})(1.20 \text{ m/s})}{42.00 \text{ kg}} \\
 &= 1.26 \text{ m/s in the same direction as he was riding}
 \end{aligned}$$

Module 9 continued

- 39. In-line Skating** Diego and Keshia are on in-line skates. They stand face-to-face and then push each other away with their hands. Diego has a mass of 90.0 kg, and Keshia has a mass of 60.0 kg. (Level 3)

- a. Sketch the event, identifying the before and after situations, and set up a coordinate axis.

Define Diego and Keshia as a closed, isolated system.

Before: $m_K = 60.0 \text{ kg}$

$m_D = 90.0 \text{ kg}$

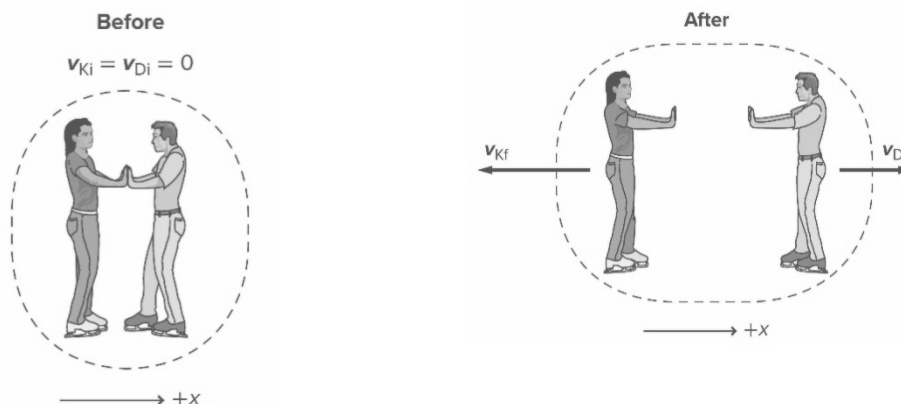
$v_i = 0.0 \text{ m/s}$

After: $m_K = 60.0 \text{ kg}$

$m_D = 90.0 \text{ kg}$

$v_{Kf} = ?$

$v_{Df} = ?$



- b. Find the ratio of the skaters' velocities just after their hands lose contact.

$$p_{Ki} + p_{Di} = 0.0 \text{ kg}\cdot\text{m/s} = p_{Kf} + p_{Df}$$

$$\text{So, } m_K v_{Kf} + m_D v_{Df} = 0.0 \text{ kg}\cdot\text{m/s} \text{ and } m_K v_{Kf} = -m_D v_{Df}$$

Thus the ratios of the velocities are

$$\frac{v_{Kf}}{v_{Df}} = -\left(\frac{m_D}{m_K}\right) = -\left(\frac{90.0\text{kg}}{60.0\text{kg}}\right) = -1.50$$

The negative sign shows that the velocities are in opposite directions.

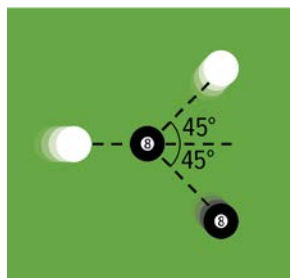
- c. Which skater has the greater speed?

Keshia, who has the smaller mass, has the greater speed.

- d. Which skater pushed harder?

The forces were equal and opposite.

- 40. Billiards** A cue ball, with mass 0.16 kg, rolling at 4.0 m/s, hits a stationary eight ball of similar mass. If the cue ball travels 45° to the left of its original path and the eight ball travels 45° in the opposite direction as shown, what is the velocity of each ball after the collision? (Level 2)



Module 9 continued

Define the two balls as a closed, isolated system. We can get momentum equations from the vector diagram.

$$p_{Cf} = p_{Ci} \sin 45^\circ$$

$$m_C v_{Cf} = m_C v_{Ci} \sin 45^\circ$$

$$v_{Cf} = v_{Ci} \sin 45^\circ = (4.0 \text{ m/s})(\sin 45^\circ) = 2.8 \text{ m/s}$$

For the eight ball,

$$p_{8f} = p_{Ci} \cos 45^\circ$$

$$m_8 v_{8f} = m_C v_{Ci} (\cos 45^\circ)$$

where $m_8 = m_C$. Thus,

$$v_{8f} = v_{Ci} \cos 45^\circ = (4.0 \text{ m/s})(\cos 45^\circ) = 2.8 \text{ m/s}$$

41. A 2575-kg van runs into the back of an 825-kg compact car at rest. They move off together at 8.5 m/s. Assuming that the friction with the road is negligible, calculate the initial speed of the van. (Level 2)

Define the van and the compact car as a closed, isolated system.

$$p_{Ci} + p_{Di} = p_{Cf} + p_{Df}$$

$$m_C v_{Ci} = (m_C + m_D) v_f$$

$$\text{so, } v_{Ci} = \frac{m_C + m_D}{m_C} v_f$$

$$v_f = \frac{(2575 \text{ kg} + 825 \text{ kg})(8.5 \text{ m/s})}{2575 \text{ kg}}$$
$$= 11 \text{ m/s}$$

42. A 0.200-kg plastic ball has a forward velocity of 0.30 m/s. It collides with a second plastic ball of mass 0.100 kg, which is moving along the same line at a speed of 0.10 m/s. After the collision, both balls continue moving in the same, original direction. The speed of the 0.100-kg ball is 0.26 m/s. What is the new velocity of the 0.200-kg ball? (Level 3)

Define the balls as a closed, isolated system.

$$m_C v_{Ci} + m_D v_{Di} = m_C v_{Cf} + m_D v_{Df}$$

$$\text{so, } v_{Cf} = \frac{m_C v_{Ci} + m_D v_{Di} - m_D v_{Df}}{m_C}$$

$$= \frac{(0.200 \text{ kg})(0.30 \text{ m/s}) + (0.100 \text{ kg})(0.10 \text{ m/s}) - (0.100 \text{ kg})(0.26 \text{ m/s})}{0.200 \text{ kg}}$$
$$= 0.22 \text{ m/s in the original direction.}$$

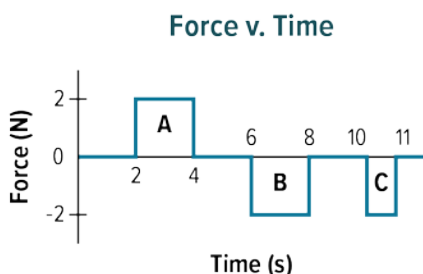
Applying Concepts

43. Explain the concept of impulse using physical ideas rather than mathematics.

A force, F , exerted on an object over a time, Δt , causes the momentum of the object to change by the quantity $F\Delta t$.

Module 9 continued

44. An object initially at rest experiences the impulses described by the graph. Describe the object's motion after impulses A, B, and C.



After time A, the object moves with a constant, positive velocity. After time B, the object is at rest. After time C, the object moves with a constant, negative velocity.

45. Is it possible for an object to obtain a larger impulse from a smaller force than it does from a larger force? Explain.

Yes, if the smaller force acts for a long enough time, it can provide a larger impulse.

46. **Foul Ball** You are sitting at a baseball game when a foul ball comes in your direction. You prepare to catch it bare-handed. To catch it safely, should you move your hands toward the ball, hold them still, or move them in the same direction as the moving ball? Explain.

You should move your hands in the same direction the ball is traveling to increase the time of the collision, thereby reducing the force.

47. A 0.11-g bullet leaves a pistol at 323 m/s, while a similar bullet leaves a rifle at 396 m/s. Explain the difference in exit speeds of the two bullets, assuming that the forces exerted on the bullets by the expanding gases have the same magnitude.

The bullet is in the rifle a longer time, so the momentum it gains is larger.

48. During a space walk, the tether connecting an astronaut to the spaceship breaks. Using a gas pistol, the astronaut manages to get back to the ship. Use the language of the impulse-momentum theorem and a diagram to explain why this method was effective.

When the gas pistol is fired in the opposite direction, it provides the impulse needed to move the astronaut toward the spaceship.

49. **Tennis Ball** As a tennis ball bounces off a wall, its momentum is reversed. Explain this action in terms of the law of conservation of momentum. Define the system and draw a diagram as a part of your explanation.

Consider the system to be the ball, the wall, and Earth. The wall and Earth gain some momentum in the collision.

50. Two trucks that appear to be identical collide on an icy road. One was originally at rest. The trucks are stuck together and move at more than half the original speed of the moving truck. What can you conclude about the contents of the two trucks?

Consider the two trucks to be a closed, isolated system. If the two trucks had equal masses, they would have moved off at half the speed of the moving truck. Thus, the moving truck must have had a more massive load.

51. **Bullets** Two bullets of equal mass are shot at equal speeds at equal blocks of wood on a smooth ice rink. One bullet, made of rubber, bounces off of the wood. The other bullet, made of aluminum, burrows into the wood. In which case does the block of wood move faster? Explain.

In each case, consider the bullet and the block of wood to be a closed, isolated system. Momentum is conserved, so the momentum of the block and bullet after the collision equals the momentum of the bullet before the collision. The rubber bullet has a negative momentum after impact, with respect to the block, so the block's momentum must be greater in this case.

Module 9 continued

Mixed Review

52. A constant force of 6.00 N acts on a 3.00-kg object for 10.0 s. What are the changes in the object's momentum and velocity? (Level 1)

The change in momentum is

$$\begin{aligned}\Delta p &= F\Delta t \\ &= (6.00 \text{ N})(10.0 \text{ s}) \\ &= 60.0 \text{ N}\cdot\text{s} = 60.0 \text{ kg}\cdot\text{m/s}\end{aligned}$$

The change in velocity is found from the impulse.

$$\begin{aligned}F\Delta t &= m\Delta v \\ \Delta v &= \frac{F\Delta t}{m} \\ &= \frac{(6.00 \text{ N})(10.0 \text{ s})}{3.00 \text{ kg}} \\ &= 20.0 \text{ m/s}\end{aligned}$$

53. An external, constant force changes the speed of a 625-kg car from 10.0 m/s to 44.0 m/s in 68.0 s. (Level 1)

- a. What is the car's change in momentum?

$$\begin{aligned}\Delta p &= m\Delta v = m(v_f - v_i) \\ &= (625 \text{ kg})(44.0 \text{ m/s} - 10.0 \text{ m/s}) \\ &= 2.12 \times 10^4 \text{ kg}\cdot\text{m/s}\end{aligned}$$

- b. What is the magnitude of the force?

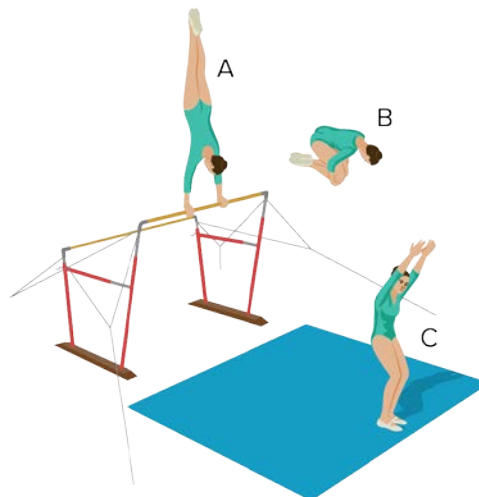
$$\begin{aligned}F\Delta t &= m\Delta v \\ F &= \frac{m\Delta v}{\Delta t} \\ &= \frac{m(v_f - v_i)}{\Delta t} \\ &= \frac{(625 \text{ kg})(44.0 \text{ m/s} - 10.0 \text{ m/s})}{68.0 \text{ s}} \\ &= 313 \text{ N}\end{aligned}$$

55. **Dragster** An 845-kg dragster accelerates on a race track from rest to 100.0 km/h in 0.90 s. (Level 1)

- a. What is the change in momentum of the dragster?

$$\begin{aligned}\Delta p &= m(v_f - v_i) \\ &= (845 \text{ kg})(100.0 \text{ km/h} - 0.0 \text{ km/h}) \left(\frac{1000 \text{ m}}{1 \text{ km}} \right) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \\ &= 2.35 \times 10^4 \text{ kg}\cdot\text{m/s}\end{aligned}$$

54. **Gymnastics** The image shows a gymnast performing a routine. First, she does giant swings on the upper bar, holding her body straight and pivoting around her hands. Then, she lets go of the high bar and grabs her knees with her hands in the tuck position. Finally, she straightens up and lands on her feet. (Level 2)



- a. In the second and final parts of the gymnast's routine, around what axis does she spin?

She spins around the center of mass of her body, first in the tuck position and then also as she straightens out.

- b. Rank in order, from greatest to least, her moments of inertia for the three positions.

giant swing (greatest), straight, tuck (least)

- c. Rank in order, from greatest to least, her angular velocities in the three positions.

B (greatest), C, A (least)

Module 9 continued

- b. What is the average force exerted on the dragster?

$$\begin{aligned}
 F &= \frac{m(v_f - v_i)}{\Delta t} \\
 &= \frac{(845 \text{ kg})(100.0 \text{ km/h} - 0.0 \text{ km/h})}{0.90 \text{ s}} \left(\frac{1000 \text{ m}}{1 \text{ km}} \right) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \\
 &= 2.6 \times 10^4 \text{ N}
 \end{aligned}$$

- c. What exerts that force?

The force is exerted by the track through friction.

56. **Ice Hockey** A 0.115-kg hockey puck, moving at 35.0 m/s, strikes a 0.365-kg jacket that is thrown onto the ice by a fan of a certain hockey team. The puck and jacket slide off together. Find their velocity. (Level 2)

Consider the hockey puck and the jacket to be a closed, isolated system.

$$\begin{aligned}
 m_p v_{pi} &= (m_p + m_j) v_f \\
 v_f &= \frac{m_p v_{pi}}{m_p + m_j} = \frac{(0.115 \text{ kg})(35.0 \text{ m/s})}{(0.115 \text{ kg} + 0.365 \text{ kg})} = 8.39 \text{ m/s}
 \end{aligned}$$

57. A 50.0-kg woman, riding on a 10.0-kg cart, is moving east at 5.0 m/s. The woman jumps off the front of the cart and lands on the ground at 7.0 m/s eastward, relative to the ground. (Level 2)

- a. Sketch the before and after situations, and assign a coordinate axis to them.

Consider the woman and the cart to be a closed, isolated system.

Before:

$$m_w = 50.0 \text{ kg}$$

$$m_c = 10.0 \text{ kg}$$

$$v_i = 5.0 \text{ m/s}$$

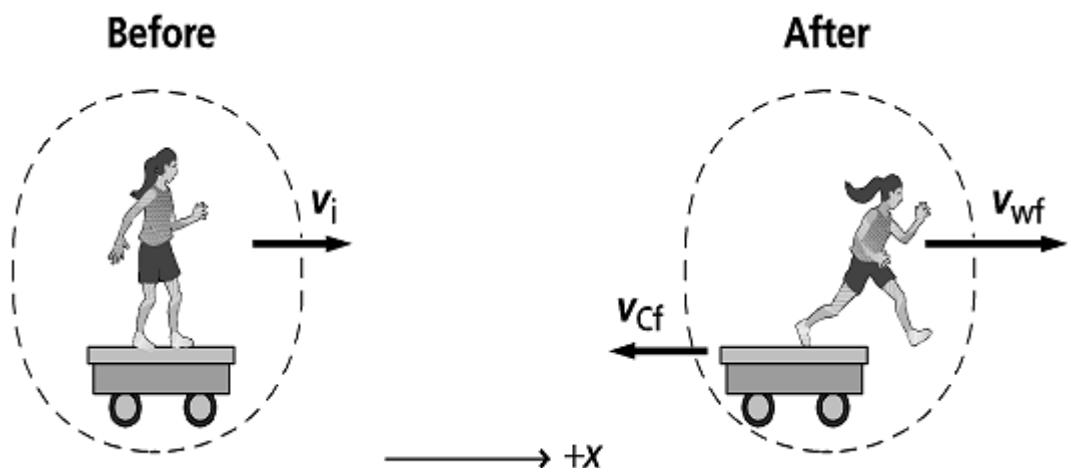
After:

$$m_w = 50.0 \text{ kg}$$

$$m_c = 10.0 \text{ kg}$$

$$v_{wf} = 7.0 \text{ m/s}$$

$$v_{cf} = ?$$



Module 9 continued

- b. Find the cart's velocity after the woman jumps off.

$$(m_w + m_c)v_i = m_w v_{wf} + m_c v_{cf}$$

$$\begin{aligned}\text{so, } v_{cf} &= \frac{(m_w + m_c)v_i - m_w v_{wf}}{m_c} \\ &= \frac{(50.0 \text{ kg} + 10.0 \text{ kg})(5.0 \text{ m/s}) - (50.0 \text{ kg})(7.0 \text{ m/s})}{10.0 \text{ kg}} \\ &= -5.0 \text{ m/s, or } 5.0 \text{ m/s west}\end{aligned}$$

58. A 60.0-kg dancer leaps 0.32 m high. (Level 3)

- a. With what momentum does he reach the ground?

Let the positive direction be upward.

Let: v_{top} = the velocity at the top of his leap.

v_i = the velocity just as he starts to leap

v_f = the velocity just as he reaches the ground again

$v_{\text{stop}} = 0 \text{ m/s}$ = the velocity after he comes to a complete stop at the ground

$$v_{\text{top}}^2 = v_i^2 + 2a\Delta x, \text{ where } a \text{ is } -9.8 \text{ N/kg and } v_{\text{top}} = 0 \text{ m/s.}$$

$$v_i = \sqrt{-2a\Delta x}$$

Just as he reaches the ground again $v_f = v_i$. His momentum, then, is:

$$\begin{aligned}p &= mv_f = mv_i = m\sqrt{-2a\Delta x} = (60.0 \text{ kg})\sqrt{-(2)(-9.8 \text{ N/kg})(0.32 \text{ m})} \\ &= 1.5 \times 10^2 \text{ kg}\cdot\text{m/s downward}\end{aligned}$$

- b. What impulse is needed to stop the dancer?

$$F\Delta t = m\Delta v = m(v_{\text{stop}} - v_f)$$

To stop the dancer, $v_{\text{stop}} = 0$. Thus,

$$F\Delta t = -mv_f = -p = -1.5 \times 10^2 \text{ kg}\cdot\text{m/s upward}$$

- c. As the dancer lands, his knees bend, lengthening the stopping time to 0.050 s. Find the average force exerted on the dancer's body.

$$F\Delta t = m\Delta v = m\sqrt{2a\Delta x}$$

$$F = \frac{m\sqrt{2a\Delta x}}{\Delta t} = \frac{(60.0 \text{ kg})\sqrt{(2)(9.8 \text{ N/kg})(0.32 \text{ m})}}{0.050 \text{ s}} = 3.0 \times 10^3 \text{ N}$$

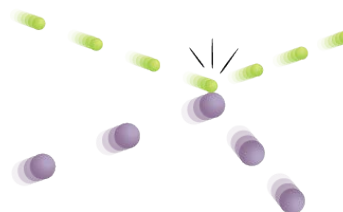
- d. Compare the stopping force with his weight.

$$\begin{aligned}F_g &= m_g = (60.0 \text{ kg})(9.8 \text{ N/kg}) \\ &= 5.88 \times 10^2 \text{ N}\end{aligned}$$

The force is about five times the weight.

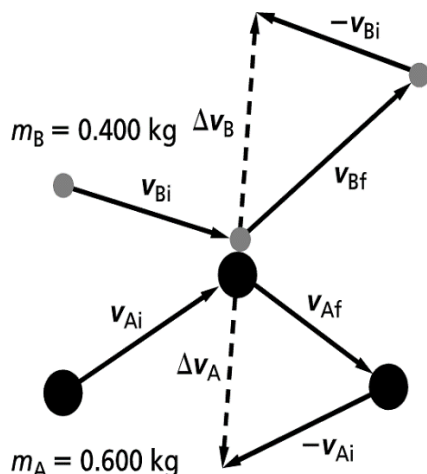
Thinking Critically

59. **Analyze and Conclude** Two balls during a collision are shown in the image, which is drawn to scale. The balls enter from the left of the diagram, collide, and then bounce away. The heavier ball, at the bottom of the diagram, has a mass of 0.600 kg, and the other has a mass of 0.400 kg. Using a vector diagram, determine whether momentum is conserved in this collision. Explain any difference in the momentum of the system before and after the collision.



Module 9 continued

Consider the two balls to be a closed, isolated system. Dotted lines show that the changes of momentum for each ball are equal and opposite: $\Delta(m_A v_A) = \Delta(m_B v_B)$. Because the masses have a 3:2 ratio, a 2:3 ratio of velocity changes will compensate.



60. **Analyze and Conclude** A student, holding a bicycle wheel with its axis vertical, sits on a stool that can rotate with negligible friction. She uses her hand to get the wheel spinning. Would you expect the student and stool to turn? If so, in which direction? Explain.

The student and the stool would spin slowly in the direction opposite to that of the wheel. Without friction there are no external torques. Thus, the angular momentum of the system is not changed. The angular momentum of the student and stool must be equal and opposite to the angular momentum of the spinning wheel.

61. **Apply Concepts** A 92-kg fullback, running at a speed of 5.0 m/s, attempts to dive directly across the goal line for a touchdown. Just as he reaches the line, he is met head-on in midair by two 75-kg linebackers, both moving in the direction opposite the fullback. One is moving at 2.0 m/s, and the other at 4.0 m/s. They all become entangled as one mass.

- a. Sketch the before and after situations.

Consider the football players to be a closed, isolated system.

Before: $m_A = 92 \text{ kg}$

$m_B = 75 \text{ kg}$

$m_C = 75 \text{ kg}$

$v_{Ai} = 5.0 \text{ m/s}$

$v_{Bi} = -2.0 \text{ m/s}$

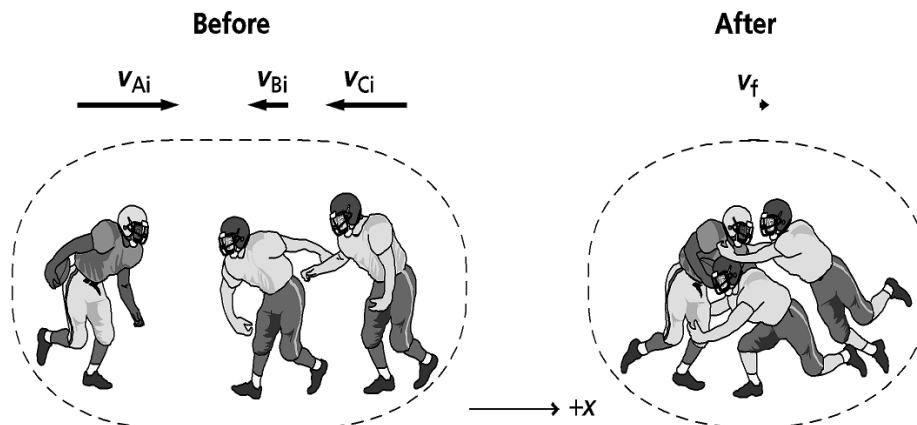
$v_{Ci} = -4.0 \text{ m/s}$

After: $m_A = 92 \text{ kg}$

$m_B = 75 \text{ kg}$

$m_C = 75 \text{ kg}$

$v_f = ?$



Module 9 continued

- b. What is the players' velocity after the collision?

$$\begin{aligned}p_{Ai} + p_{Bi} + p_{Ci} &= p_{Af} + p_{Bf} + p_{Cf} \\m_A v_{Ai} + m_B v_{Bi} + m_C v_{Ci} &= m_A v_{Af} + m_B v_{Bf} + m_C v_{Cf} \\&= (m_A + m_B + m_C) v_f \\v_f &= \frac{m_A v_{Ai} + m_B v_{Bi} + m_C v_{Ci}}{m_A + m_B + m_C} \\&= \frac{(92 \text{ kg})(5.0 \text{ m/s}) + (75 \text{ kg})(-2.0 \text{ m/s}) + (75 \text{ kg})(-4.0 \text{ m/s})}{92 \text{ kg} + 75 \text{ kg} + 75 \text{ kg}} \\&= 0.041 \text{ m/s}\end{aligned}$$

- c. Does the fullback score a touchdown?

Yes. The velocity is positive, so the football crosses the goal line for a touchdown.

Writing in Physics

62. How can highway barriers be designed to be more effective in saving people's lives? Research this issue and describe how impulse and change in momentum can be used to analyze barrier designs.

The change in a car's momentum does not depend on how it is brought to a stop. Thus, the impulse also does not change. To reduce the force, the time over which a car is stopped must be increased. Using barriers that can extend the time it takes to stop a car will reduce the force. Flexible, plastic containers filled with sand often are used.

63. While air bags save many lives, they also have caused injuries and even death. Research the arguments and responses of automobile makers to this statement. Determine whether the problems involve impulse and momentum or other issues.

There are two ways an air bag reduces injury. First, an air bag extends the time over which the impulse acts, thereby reducing the force. Second, an air bag spreads the force over a larger area, thereby reducing the pressure. Thus, the injuries due to forces from small objects are reduced. The dangers of air bags mostly center on the fact that air bags must be inflated very rapidly. The surface of an air bag can approach the passenger at speeds of up to 322 km/h (200 mph). Injuries can occur when the moving bag collides with the person.

Systems are being developed that will adjust the rate at which gases fill the air bags to match the size of the person.